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The Competence of Sub-Optimal Theories of Structure Mapping on Hard Analogies

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Abstract: Structure-mapping is a provably NP-Hard problem which is argued to lie at the core of the human metaphoric and analogical reasoning faculties. This NP-Hardness has meant that earlier naïve attempts at optimal solutions to the problem have had to be augmented with sub-optimal heuristics to ensure tractable performance. This paper considers various epistemological grounds for qualifying the competence of such heuristic approaches, and offers a quantitative evaluation of the sub-optimal performance of three different models of analogy/metaphor, SME, ACME and Sapper.

1. Introduction

Metaphor interpretation and Analogical reasoning are two, closely related, cognitive faculties which rely upon a structure mapping process to generate coherent and systematic correspondences between two domains of discourse. But inasmuch as structure-mapping is essentially a *graph-isomorphism process* which must consider a combinatorial number of such correspondences to generate an optimal mapping, it is intuitively an NP-hard problem.

A variety of computational approaches to the problem have been described in the AI literature, such as the *Structure Mapping Engine* (SME) of [Falkenhainer *et al*. 1989], the *Analogical Constraint Mapping Engine* (ACME) of [Holyoak & Thagard 1989] and the *Sapper* model of [Veale *et al* 1996a,b]. However, only the first of these models, SME, has ever attempted to provide an optimal solution to the problem, but the combinatorial explosion that occurs in some problem representations has moved its designers to advocate a sub-optimal greedy-merge approach (see [Oblinger & Forbus 1990]) and later, an incremental approach (see [Forbus *et al.* 1994]). Although a basic complexity analysis of SME is provided by [Falkenhainer *et al.* 1989], no qualitative, epistemological criteria have been offered to identify those areas of operation that would lead to such factorial explosion. At its heart the original SME is a forest-matching mechanism, which extends known results regarding the $O(N^2)$ complexity of determining sub-tree isomorphism (e.g., see [Akutsu, 1992]) to forests of inter-tangled tree representations. Layered on top of this forest matcher is a factorial merge process which combines the results of the polynomial sub-tree matching phase (called partial maps, or *pmaps*) into larger, global mappings (called *gmaps*). This merge process is clearly O(2^N) where N is the number of pmaps (isomorphic sub-tree matches) involved. SME's designers essentially state in their analysis that any representation that causes N to be large will cause SME to be overly factorial, but do not identify any particularly important domain of discourse where this will necessarily be so.

However, one such important domain of metaphoric and analogical concern is identified in [Veale *et al.* 1996b], who demonstrate empirically that object-centered representations (for noun-based concepts such as Composer, War, etc.) tend to comprise a multitude of highly-connected shallow trees rather than a small number of deeply-nested trees. A mapping between two such domains is illustrated in Figure 2.

Such domains as these, which underlie a good deal of metaphoric discourse, therefore exacerbate SME's original $O(2^N)$ complexity. For example, the metaphor *Surgeons are Butchers* requires less that 15 seconds of processing time in Sapper, yet generates enough pmaps to keep an optimal SME busy for many billions of years. Veale *et al*. report the following surprising results of Figure 1 for an empirical test involving over 100 such object-centered representations (as drawn from the domain of professions):

Aspect	Optimal-SME	ACME	Sapper
Avg.# pmaps	386 (per metaphor)	12,657	18
Avg. time	N/A - worst case O(2^{386}) seconds	N/A	12.5 seconds

Figure 1: Comparative evaluation of Sapper, SME and ACME. The unavailability of time figures for SME and ACME reflects the inability of these models to generate a result in a matter of days.

Sapper out-performs SME in these domains because it is designed to seek out structure *laterally* from shallow trees that are connected via common elements, while SME seeks structure vertically from the hierarchical nesting of deep trees. This paper builds upon these results to show that while heuristic, sub-optimal *greedy-SME* and *Incremental-SME* are not so factorially spendthrift in their time performance, they are still very sensitive to tree organization, so their quality of mapping is nevertheless greatly diminished when dealing with object-centered representations.

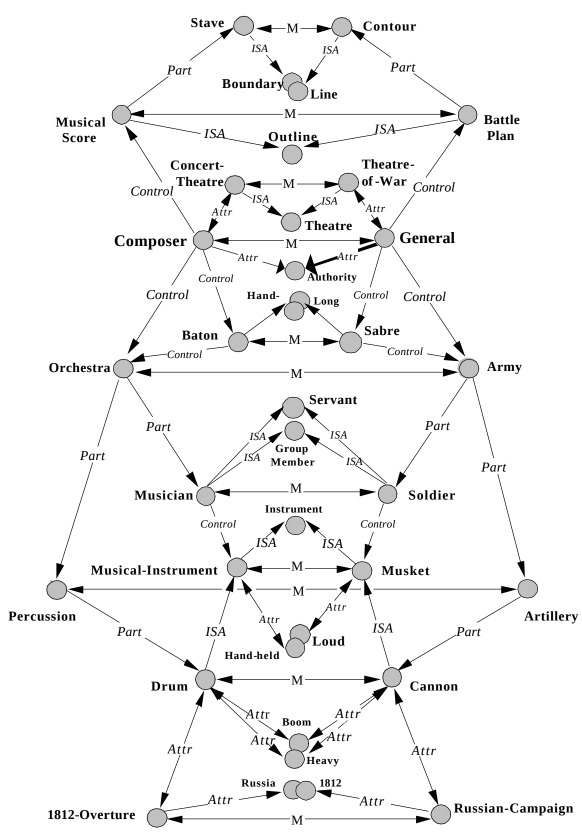

Figure 2: Partial domain descriptions relating to the concepts `Composer` *and* `General`*.*

We also apply our intuitions to the Sapper and ACME models, demonstrating that while the latter is from the outset a sub-optimal model, it too exhibits worrying lapses of competence on hard problems. This discussion will also allow us to outline in greater detail exactly what we mean by a *hard* analogical problem.

2. Cognitive Theories of Structure Mapping

ACME approaches the structure-mapping problem from a different perspective than either SME or Sapper, pursuing what might be called a *natural computation approach* to analogy and metaphor. ACME models structure-mapping as a problem of parallel constraint satisfaction, in which the demands of 1-to-1 coherence and structural systematicity are coded as soft constraints, or pressures, on the system. Ultimately, it is a sub-optimal approach which offers no guarantees as the quality of its resultant mapping.

ACME employs a Hopfield-style connectionist network to encode mapping constraints (see [Hopfield and Tank, 1985]). Every structure-mapping hypothesis— either between a source and target predicate or between a source and target entity— is coded as a distinct neuron. Likewise, structural entailments among these hypotheses are coded as bi-directional excitatory links between the corresponding nodes, while inhibitory links are used to connect mutually exclusive hypotheses.

Interestingly, such an arrangement can be seen as the connectionist equivalent of a 2-CNF SAT formula, which raises the question of ACME's logical soundness. Indeed, it happens that the use of bi-directional linkages in ACME — which makes all implications *mutual* implications — means that an ACME representation is logically unsound. Consider an example mapping of $P(X, Y)$ to $G(A, B)$: not only does $P{:}G \rightarrow X{:}A$ and $P{:}G \rightarrow Y{:}B$, the reciprocal implications $X{:}A \rightarrow P{:}G$ and $Y{:}B \rightarrow P{:}G$ also hold. Yet, an isomorphic analogy might allowably map $X{:}A$ but not $P{:}G$; the sound reciprocal implication is $X{:}A \wedge Y{:}B \rightarrow P{:}G$, or the 3-CNF formula $(\neg X{:}A \vee \neg Y{:}B \vee P{:}G)$. ACME is sound only when the source and target structures are trees (hence argument mappings *do* imply predicate mappings), but as already noted, structure mapping is polynomially bounded anyway in such situations.

Overall, the complexity prognosis of ACME is not good: as a feedback-based neural network, there is no guaranteed polynomial bound on its time performance. Yet, because the network size is polynomially-bounded (i.e., $O(n^2)$ nodes and $O(n^4)$ linkages, where n is the number of distinct symbols in the source domain), the theoretical results of [Bruck & Goodman 1990] apply, who prove that a Hopfield-style network of polynomial size can only optimally solve NP-hard problems if $NP = P$. This begs the question, then, if an ACME network can only realistically embody a polynomial algorithm, why should it be allowed to consume an exponential amount of time doing so?

2.1. Sapper: A Memory-Situated Model

The Sapper model of [Veale *et al.* 1996a,b] views semantic memory as a localist graph in which nodes represent distinct concepts, and arcs between those nodes represent semantic / conceptual relations between those concepts. Memory management under Sapper is pro-active toward structure mapping, that is, it employs rules of structural similarity to determine whether any two given nodes may at some future time be placed in systematic correspondence in a metaphoric context. If so, Sapper notes this fact by laying down a *bridge relation* between these nodes, to be exploited in some future structure-mapping session. The two heuristics which Sapper employs to lay down these bridges are termed Triangulation and Squaring:

> **Triangulation:** *If memory already contains two linkages L_{ij} and L_{kj} of semantic type L forming two sides of a triangle between the concept nodes C_k, C_i and C_j, then complete the triangle and augment memory with a new conceptual bridge linkage B_{ik}.*

> **Squaring:** *If B_{jk} is a bridge, and if there already exists the linkages L_{ij} and L_{lk} of the semantic type L, forming three sides of a square between the concept nodes C_i, C_j, C_k and C_l, then complete the square and augment memory with a new bridge linkage B_{il}.*

At some future time, if Sapper wishes to determine a structural mapping between a target domain rooted in the concept node T (for Target) and one rooted in the node S (Source), it applies the algorithm of Figure 3.

The Sapper algorithm comprises two main phases: the first of these seeks out the set Φ of all well-formed and balanced semantic pathways (of length $\leq 2H$) that originate at the root node of the target (T), and terminate at the root node of the source (S), crossing a single conceptual bridge (i.e., the domain cross-over point) at its mid-point. Each such pathway corresponds to a partial interpretation (a pmap in SME parlance) of the metaphor/analogy. The second phase coalesces this collection of pmaps Φ into a coherent global whole; it does this using a *seeding algorithm* (see [Keane and Brayshaw, 1988]) which starts with the structurally richest pmap Γ as its seed, and then attempts to fold each other pmap into this seed, if it is coherent to do so, in descending order of the richness of those pmaps. This seeding phase is directly equivalent to the greedy merge phase of Greedy-SME (see [Oblinger & Forbus 1990]), which amends the original SME design.

```
Spread Activation from nodes (T)arget and (S)ource in memory to a horizon H
   When a wave of activation from T meets a wave from S at a bridge T′:S′
          linking the tenor domain concept T′ to the vehicle domain concept S′
Then:
      Find a path of semantic relations R that links both T′ to T and S′ to S
      If R is found, then the bridge T′:S′ is balanced relative to T:S, so Do:
        | Generate a partial interpretation (pmap) π of the metaphor T:S as follows:
        |      For every tenor concept t between T′ and T as linked by R Do:
        |        Put t in alignment with the equivalent concept s between S′ and S
        |        π ← π ∪ {<t : s>}
        | Φ ← Φ ∪ {π}
   Once the set Φ of all pmaps within the horizon H have been found, Do
      Evaluate the richness of each pmap π ∈ Φ
      Sort the collection Φ of pmaps in descending order of richness.
      Pick the first (richest) interpretation Γ ∈ Φ as a seed:
      Visit every other pmap π ∈ (Φ - Γ) in descending order of richness
         If it is coherent to merge π with Γ (i.e., without violating 1-to-1ness) then
              Γ ← Γ ∪ π
         Otherwise discard π
      When Φ is exhausted, Γ will contain the overall Sapper interpretation of T:S
```

Figure 3: The Sapper Algorithm, as based on the exploitation of cross-domain bridge-points in semantic memory.

3. Proof: Structure-Mapping is NP-Hard

In this section we place our arguments on a solid footing by proving the NP-Hardness of the structure mapping problem. Though the known NP-complete problem LCS (Largest Common Sub-Graph) is perhaps a more immediate match, we instead employ here 3DM (3-Dimensional Matching) as a proof basis, a problem which seeks to obtain a non-overlapping matching of points in a 3-D space. A consideration of 3DM will shed light on the worst case scenario as encountered by the greedy heuristics employed by greedy-SME. To begin with, 3DM is defined as follows (from [Garey and Johnson, 1979]):

> ***Unique 3-Dimensional Matching (3DM):*** *Given a set M of points in 3-D space, i.e., $M \subseteq X\times Y\times Z$, where X, Y and Z are disjoint sets of integers and $|X|=|Y|=|Z| = q$, find the largest set $M' \subseteq M$ such that no two elements of M' agree in any coordinate.*

This problem is a Cartesian variant of the well-known *N-Queens* problem, in which one must place n queens on an $n\times n$ board such that no two queens occupy the same rank, file or diagonal. The point compatibility issue is also very much like the 1-to-1 isomorphism constraint on cross-domain structure mappings.

Proof: To reformulate 3DM as a problem of structure-mapping, it is necessary to represent each 3-D point $<X_i, Y_i, Z_i> \in M$ as a pair of predicates, one in each of the source S and target T domains, such that these predicates are only allowed to map onto each other. Furthermore, it is required that any isomorphic mapping must not contain two different predicate matches that arise from two points that share one or more coordinates. We can ensure this using the following polynomial transformation:

$\forall < X_i, Y_i, Z_i > \subseteq M$ Do

add PXYZ(X_i, Z_i) to S

and add PXYZ (Y_i, $\Omega * X_i + Y_i$) to T

where $\Omega = \max(\max(X\cup Y\cup Z), |\min(X\cup Y\cup Z)|)$

Now, because the predicate P is uniquely tagged with the subscript XYZ which ties it to a particular 3-D point, these two predicate structures can map only to each other. When so mapped during the analogy process, such a map results in the creation of the following structure (a *pmap* in SME parlance):

$$map_i = \{< X_i, Y_i >, < Z_i, \Omega * X_i + Y_i >\}$$

In this manner a root mapping will be created for each point in M. Note also that $\Omega * X_i + Y_i$ is unique for each pairing of X_i and Y_i, thus X_i, Y_i and Z_i are *tied together* and cannot be cross-mapped with any other point coordinate. Suppose we have two such pmaps, $map_i = \{< X_i, Y_i >, < Z_i, \Omega * Xi + Yi>\}$ and $map_k = \{<X_k, Y_k>, <Z_k, \Omega * X_k + Y_k>\}$, arising out of the two points $<X_i, Y_i, Z_i>$ and $<X_k, Y_k, Z_i>$ which share a Z-coordinate Z_i. These maps cannot therefore be merged together to create a larger mapping because such a merge would result in Z_i being mapped to both $\Omega * X_i + Y_i$ and $\Omega * X_k + Y_k$, which is a violation of mapping isomorphism. The same constraint can also be demonstrated for points sharing either an X or Y coordinate.

Once a maximal isomorphic mapping structure (a gmap) is found for the analogy, each pair $<X_i, Y_i>$ and $<Z_i, \Omega * X_i + Y_i>$ of this gmap can then be decomposed and reassembled (in polynomial time) to recreate the point $<X_i, Y_i, Z_i>$ which is then added to M'. Since the gmap is maximal, so is M'. Because it solves 3DM, structure-mapping is thus NP-Hard. ☐

4. Problem Reorganization for Tractability

Though the provable NP-Hardness of structure-mapping precludes any generally optimal solution to the problem, a large body of problem instances may nevertheless be tractably amenable to an optimal Sapper variant. In particular, if an optimal Sapper solution can be obtained for a large enough body of problem examples, these solutions can be used as ceilings against which to measure the competence of sub-optimal heuristics like greedy merging/seeding.

The domain descriptions in the Sapper profession corpus contain on average over 120 predications each. Test metaphors in the profession corpus thus generate too many partial mappings to make optimal evaluation tractable. Yet, some problem re-organization can be applied to reduce the number of pmaps to frequently an Optimal-Sapper feasible, without losing the combinatorial scope of the interpretation. This reorganization process, whereby redundant areas of the combinatorial search space are pruned away, is the analogical equivalent of performing *arc-consistency tests* in satisfaction problems to a priori remove contradictory variable assignments (see [Mohr and Henderson, 1986]).

For each metaphor (whose pmap set is denoted Φ) a conflict graph is constructed in $O(|\Phi|^2)$ time, by determining for each pmap the set of other pmaps with which it cannot be combined. The conflict set CF_i for a particular pmap $\pi_i \in \Phi$ is thus defined as:

$$CF_i = \{\pi_k \mid k \neq i \wedge \neg systematic(\pi_i, \pi_k)\}$$

Compatibility between pmaps can thus be defined:

$$compatible(\pi_i, \pi_k) \text{ iff } CF_i \subseteq CF_k \wedge \pi_i \notin CF_k$$

With this information we can analyse the pmaps that comprise the core of the structure-mapping problem, recognize any compatibility-based redundancies, and redistribute them accordingly, as follows:

$\forall\ \pi_i\ \pi_j,\ i \neq j$, if $compatible(\pi_i, \pi_j)$ then

$\quad \forall\ \pi_k \in CF_j - CF_i$ do

$\quad \pi_k \leftarrow \pi_k \cup \pi_i$

$\quad \Phi \leftarrow \Phi - \pi_i$

Given that the combinatorial merge stage of an Optimal-Sapper algorithm is $O(2^{|\Phi|})$, each such pmap factored out a priori lowers the eventual cost another exponential notch. On our main experimental corpus of profession metaphors, we have found that problem reduction of this form reduces the number of pmaps for each metaphor by an average of 60%, pruning the search space of the most intractable instance, `Generals` *are*

`Surgeons`, from O(2^{39}) to one more manageable by Optimal-Sapper, O(2^{17}).

5. Experiment: Sapper Vs. Greedy-SME

The ability to determine an optimal mapping for each of our test metaphors allows us to quantify, in real terms, the competence of sub-optimal heuristics such as seeding and greedy-search as a percentage of optimal performance.

If Composer is like General

Then Drum is like Cannon
and Powerful is like Loud
and Loud is like Powerful
and Conductor_Baton is like Sword
and Tchaikovsky is like Napoleon
and Libretto is like Plan
and Narrow is like Dangerous
and 19th_Century is like French
and Music_Recital is like Cavalry_Charge
and Long is like Sharp
and Orchestra is like Army
and Listener is like Soldier
and W_A_Mozart is like George_Patten
and Percussion is like Artillery
and Theatre is like Influential
and Russian is like 19th_century
and Music_Composition is like Bomb_Raid
and Musical is like Healthy
and Music_Note is like Enemy_Soldier
and Sudden is like Dead
and Piano is like Snub_Fighter
and Fictional is like On_Target
and Character is like Smart_Bomb
and 18th_Century is like Arrogant
and Symphony is like Military_Propaganda
and Violin is like Musket
and Musical_Score is like Enemy_Army
and Operatic_Act is like Medal
and Opera is like Military_Uniform
and Inspiration is like Corpse

*Figure 4: Simulated Greedy-SME interpretation of "*Composers are Generals*".*

Before doing so, we should consider the nature of the mapping interpretations that structure-mapping algorithms will generate for these test metaphors. The mapping of Figure 1 previously was the Sapper interpretation of the metaphor `Composers` *are* `Generals`, while the mapping of Figure 4 is that returned by greedy-SME for the same metaphor. We note that an official implementation of greedy-SME is not yet publicly available ([Forbus, 1996]). So in lieu of an official implementation, we currently simulate greedy-SME by feeding the pmaps generated by the publicly available optimal-SME through the Sapper seeding stage, which is a computationally equivalent process.

A selection of the mappings in Figure 4 above are displayed in an italics face to convey their dubious status; these noisy mappings are essentially 'ghosts', mappings that might work in another metaphoric context but which are not systematic here. So why does greedy-SME generate so many ghosts while Sapper produces none, when both employ equivalent merge processes? The answer is that Sapper pmaps are generally much richer than those of SME. The reason for this is to be found in three tacit assumptions that underlie the seeding process: first, that a goodness ordering can be placed upon the set of pmaps; secondly, that the pmap chosen as seed for the merge is rich enough to justify its own inclusion in the global mapping; and thirdly, that this seed is rich enough to nudge the overall merge process toward a good to optimal global mapping. Greedy-SME, unlike Sapper, violates all three assumptions, because of the nature of object-centred domains; as these domains are best represented as a broad forest of many shallow trees rather than a tight forest of few, deep trees (see [Veale *et al.* 1996]), the pmaps generated by SME in object-centred metaphors are equally shallow and numerous. In fact, these pmaps resemble the geometric pmaps generated in section 3 when reposing 3DM as structure-mapping. One would not expect a greedy approach to work in this context as no one pmap has enough structure to successfully guide the merge process; the same is true here of metaphor.

To quantify these intuitions, the competence of both Sapper and greedy-SME has been experimentally determined over the test corpus of 100+ profession metaphors. For this experiment the optimal-Sapper of section 4 is used as a savant: a mapping of a sub-optimal interpretation is considered valid if it is also contained in the optimal Sapper interpretation. The sub-optimal competence of Sapper and greedy-SME is thus calculated as

100 * (# valid mappings) / (Total # mappings).

If this validity criterion seems overly strict and *all-or-nothing*, it needs to be for tractability reasons. If one were to evaluate a noisy interpretation on the basis of its largest systematic subset, the partition of the interpretation into signal and noise would in itself would be a generally intractable problem of combinatorial dimensions. Comparative results are displayed in Figure 5 below:

Rating	Sapper	Greedy SME	Optimal	Random Sapper
Competence	95.2%	18.7%	100%	80.5%
% of times optimal	77%	0%	100%	45%

Figure 5: Comparative trials of Sapper and Sub-Optimal greedy-SME with a random control (Sapper using random seed selection).

Greedy-SME performs disappointingly on these trials, significantly trailing even the random control trial, in which a random merging of coherent Sapper pmaps is generated as an interpretation for each metaphor (i.e., random-Sapper is Sapper with a randomly ordered seed stage). These results speak for the importance of structurally rich pmaps, for if these are rich enough then even a random coalescence of pmaps will generate a good interpretation. In contrast, if the available set of pmaps are structurally impoverished, as with SME in object-centred domains, not even a best-first sorting will compensate. The random trials are thus revelatory, indicating that a system's true competence is to be found in the processes which generate pmaps, not in those which combine them.

6. Where the Hard Analogies Are

What do the results of section 5 say about the identifiable qualities of *hard* analogies/metaphors? Clearly, when employing an optimal mapping algorithm, the number of distinct roots in the forest-of-trees representation of each domain is a direct indicator of the exponential requirements of the algorithm. What can be said of the hardness of analogies as perceived by sub-optimal approaches such as Sapper and greedy-SME?

In complexity terms no problem instance is— strictly speaking— *hard* to a sub-optimal structure matcher, as the number of pmaps is largely irrelevant in a O(n2) greedy merging / seeding process. However, if one wishes to measure hardness in terms of the likelihood of generating a quality (i.e., ghost-free and accurate) interpretation, the best indicator of hardness is the average structural richness of each pmap (i.e., the average number of mappings in each pmap). The lower this average richness, the more probable it is that any two pmaps can be coherently merged, and thus the more likely that the final interpretation will be noisy and ghost-ridden. In contrast, the higher this average, the more probable it is that the final interpretation will be near optimal, and less likely to contain ghosts (as each pmap merge operation will have a greater chance of failure).

If we have side-lined ACME's sub-optimal approach to structure-mapping in this paper, it is due to the belief that ACME represents an excessive approach to the problem. Recall that ACME can be characterized as a 2-SAT problem, where network nodes mirror SAT variables, and network linkages mirror SAT clauses. From the ratio of ACME nodes to linkages for any given metaphor/analogy, we can determine the equivalent SAT ratio of clauses to variables as $O(n^2)$, thus making an ACME problem hugely over-constrained (see [Mitchell *et al.* 1992]). Given the large networks which ACME can construct for a hard problem (> 12,000 nodes), existing relaxation techniques based on constraint prioritisation do not seem practical (see [Bakker *et al*. 1993]). ACME thus reduces to a difficult subclass of *maximal 2-SAT*, with the size of that subset of clauses it must leave unsatisfied growing exponentially with the extent of network over-constraint, which itself grows quadratically with metaphor size. In this case, sub-optimality certainly thus not imply tractability.

In closing, we note that the profession corpus upon which our experiments are based is available from the following URL, in Sapper, ACME and SME formats: : *http://www.compapp.dcu.ie/~tonyv/*

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